Quiz 9, MATH 240, Fall 2023

Write your name clearly.

Name:

UID:

(1) Let $A = \begin{pmatrix} 3 & 0 & 0 \\ 1 & 4 & -3 \\ 0 & 0 & 3 \end{pmatrix}$.

(a) (7 points) Find the eigenvalues of A.

(b) (8 points) Find a **basis** of eigenvectors for the eigenspace corresponding to the **smallest eigenvalue**.

(c) (5 points) Say that you calculate an eigenvector corresponding to the largest eigenvalue to be $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. Is A diagonalizable? If not, justify your answer; if yes, give matrices P and D such that $A = PDP^{-1}$.

(a) $\det(A - xI) = \begin{vmatrix} 3-x & 0 & 0 \\ 1 & 4-x & -3 \\ 0 & 0 & 3-x \end{vmatrix}$

$= (3-x)\begin{vmatrix} 4-x & -3 \\ 0 & 3-x \end{vmatrix}$

$= (3-x)^2(4-x)$.

So the eigenvalues of A are 3 and 4.

(b) $\mathrm{Nul}(A-3I) = \mathrm{Nul}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & -3 \\ 0 & 0 & 0 \end{pmatrix}$

$= \mathrm{Span}\{(-1,1,0), (3,0,1)\}$.

(c) A is diagonalizable because for each eigenvalue, the multiplicity is equal to the dimension of its eigenspace. Indeed, $A = PDP^{-1}$, where

$D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ and $P = \begin{pmatrix} 0 & -1 & 3 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.